1996: PART A

1) $f'(x)$:

x	-3 to -2	-2 to 1	1 to 4	4 to 5
sign	$+$	$-$	$-$	$+$
	/	\	_ \	_ /

a) Rel. max @ $x = -2$ since $f'(x)$ changes sign from +ve to -ve at $x = -2$

b) Rel. min @ $x = 4$ since $f'(x)$ changes sign from -ve to +ve at $x = 4$

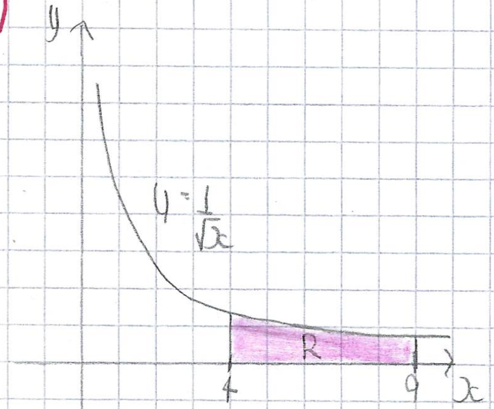
c) $f''(x)$:

x	-3 to -1	-1 to 1	1 to 3	3 to 5
sign	$-$	$+$	$-$	$+$
	$\frown$	$\smile$	$\frown$	$\smile$

concave up on $(-1, 1)$ and $(3, 5)$ since f' is increasing on these intervals

d) $f(1) = 0$
see graph

2)



a) Area $R = \int_4^9 \frac{1}{\sqrt{x}}\,dx$
$= 2$

b) $\int_4^k \frac{1}{\sqrt{x}}\,dx = \int_k^9 \frac{1}{\sqrt{x}}\,dx$

$\left[2\sqrt{x}\right]_4^k = \left[2\sqrt{x}\right]_k^9$

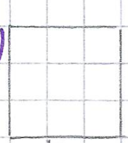
$2\sqrt{k} - 2(2) = 2\sqrt{9} - 2\sqrt{k}$

$2\sqrt{k} - 4 = 6 - 2\sqrt{k}$

$4\sqrt{k} = 10$

$\sqrt{k} = \frac{5}{2}$

$k = \frac{25}{4}$

c) 

$1/\sqrt{x}$

$$\text{volume} = \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$$
$$= \int_4^9 \left(\frac{1}{x}\right) dx$$
$$= 0.811 \text{ to 3 s.f.}$$

5) $S(t) = Ce^{kt}$

a) $S(0) = 6$

$S(0) = Ce^{0} = 6$

$C = 6$

$2C = Ce^{5k}$

$2 = e^{5k}$

$5k = \ln 2$

$k = \frac{\ln 2}{5}$

$C = 6,\ k = \frac{\ln 2}{5}$

b) $\frac{1}{13-3}\int_3^{13} 6e^{(\ln 2/5)t}\,dt$

$= \frac{1}{10}\int_3^{13} 6e^{(\ln 2/5)t}\,dt$

$= 19.680$ billions gal/yr

c) $\int_5^7 S(t)\,dt$

$= \frac{7-5}{2(4)}\left[S(5) + 2[S(5.5) + S(6) + S(6.5)] + S(7)\right]$

$= \frac{2}{8}(34.46095 + 2(37.90705 + 41.35314 + 44.79925) + 48.24533)$

$= \frac{1}{4}(330.82512)$

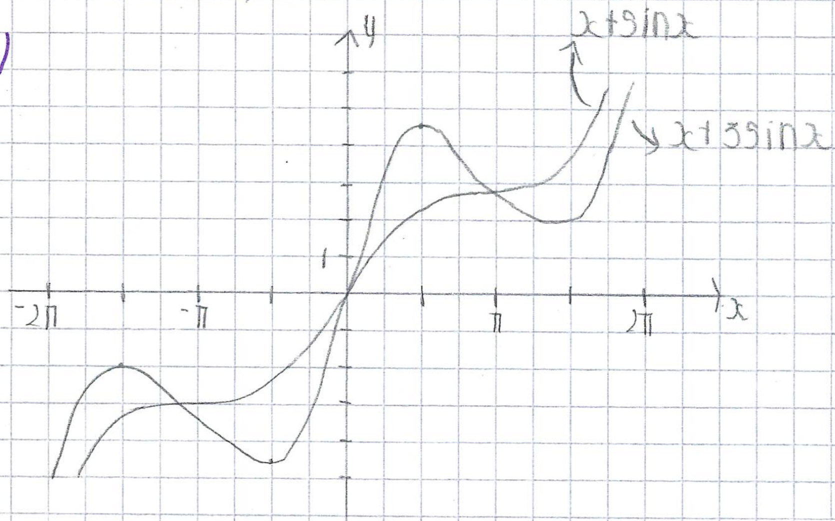
$= 20.677$

d) Total amount of coca cola consumed, in billions of gallons during the years 1985 and 1986 or from 1985 to 1987

46: PART B

4) $f(x) = x + b\sin x, \ -2\pi \le x \le 2\pi$

a)



b) $f'(x) = 1 + b\cos x$

$1 + b\cos x = 1$
$b\cos x = 0$
$\cos x = 0$
$x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$

$x + b\sin x = x + b$
$b\sin x = b$
$\sin x = 1$
$x = \frac{\pi}{2}, -\frac{3\pi}{2}$

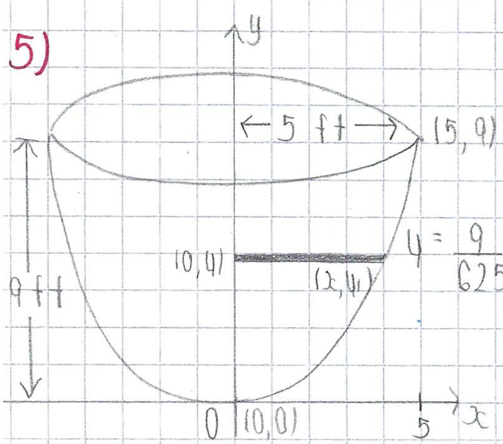
$x = -\frac{3\pi}{2}$ or $\frac{\pi}{2}$

c) No, because $f'(x) = 1$

d) $f''(x) = -b\sin x$
$f''(x) = 0$
$-b\sin x = 0$
$\sin x = 0$

$f(x) = x + b\sin x$
$= x + b(0) = x$
$\therefore$ at x coordinates of any inflection points

5)



$y = \frac{9}{625}x^4$

$y = \frac{9}{625}x^4$
$625y = 9x^4$
$x^4 = \frac{625y}{9}$
$x = \left(\frac{625y}{9}\right)^{1/4}$

a) $R(x) = x - 0 = \left(\frac{625y}{9}\right)^{1/4}$

Volume $= \pi \int_0^9 \left[\left(\frac{625y}{9}\right)^{1/4}\right]^2 dy = \pi \int_0^9 \frac{25}{3}\sqrt{y}\, dy = 471.239 \text{ ft}^3 = 150\pi \text{ ft}^3$

b) time $= \dfrac{\text{distance}}{\text{rate}} = \dfrac{150\pi}{8} = 59$

59 minutes

c) $V = \pi \displaystyle\int_0^h \frac{25}{3}\sqrt{y}\, dy$

$\dfrac{dV}{dt} = \dfrac{25}{3}\pi\sqrt{h}\,\dfrac{dh}{dt}$

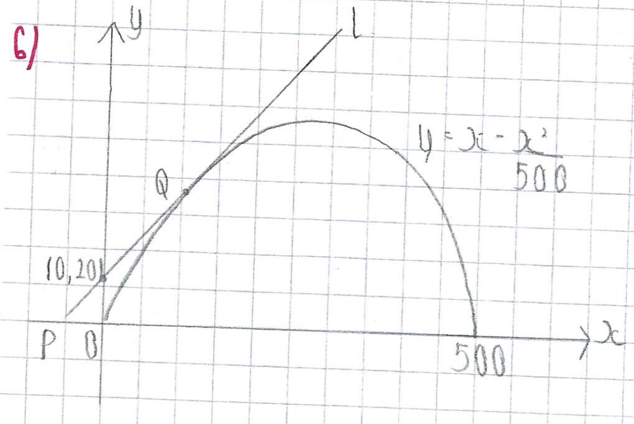
We know $\dfrac{dV}{dt} = 8$, $h = 4$

$8 = \dfrac{25}{3}\pi(2)\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{24}{50\pi}$

$\dfrac{dh}{dt} = \dfrac{12}{25\pi}$ ft/min

6)



a) $y' = 1 - \dfrac{1}{250}x$

let Q be $\left(x, x - \dfrac{x^2}{500}\right)$

slope $l = \dfrac{20 - \left(x - \frac{x^2}{500}\right)}{0 - x} = \dfrac{20 - x + \frac{x^2}{500}}{-x}$

$\therefore 1 - \dfrac{x}{250} = \dfrac{x - \frac{x^2}{500} - 20}{x}$

$1 - \dfrac{x}{250} = 1 - \dfrac{x}{500} - \dfrac{20}{x}$

$-\dfrac{x}{500} = -\dfrac{20}{x}$

$x^2 = 10000$

$x = 100$

b) slope $l = 1 - \dfrac{100}{250} = \dfrac{3}{5}$

Equation of l:

$y - 20 = \dfrac{3}{5}(x - 0)$

$y = \dfrac{3}{5}x + 20$

$$0$$
$$-\frac{x}{250} = 0$$
$$\frac{x}{250} = 1$$
$$x = 250$$

height of hill at $x = 250$:

$$y = 250 - \frac{250^2}{500} = 125 \text{ feet}$$

height of line at $x = 250$:

$$y = \frac{3}{5}(250) + 20 = 170 \text{ feet}$$

yes, the spotlight hits the tree